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THEORY OF HEAT AND MASS TRANSFER IN DISPERSE MEDIA

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The equations of heat and mass transfer in heterogeneous media obtained in [1] are generalized to the case of disperse-phase particles of arbitrary form.

In describing a series of processes in nature and engineering associated with heat or mass propagation in granular layers, the model of two coexisting continua is widely used, in the form

$$\varepsilon d_1 c_1 \left(\frac{\partial}{\partial \tau} + \mathbf{u} \cdot \nabla \right) T_1 = \lambda_* \Delta T_1 + \sigma (T_2 - T_1),$$

(1)

$$(1 - \varepsilon) d_2 c_2 \frac{\partial T_2}{\partial \tau} = \sigma (T_1 - T_2),$$

which has been repeatedly derived by different methods [2-5]. It is written here for the case of heat transfer. The quasi-steady approximation, in which the coefficients λ_* and σ may be regarded as constants, has become the most popular in practical calculations. However, even in this simplest formulation, the solution of problems is obtained in a form that is cumbersome and difficult to interpret [3, 6, 7]. In addition, the coefficients λ_*, σ are by no means always constant [1, 5]. Therefore, there is a pressing need both for the investigation of different approximate methods of solving Eq. (1) [8, 9] and for theoretical understanding of the given system and the possibility of replacing it by simpler equations [1, 10].

As is known [1], two relaxational processes occur in a heterogeneous disperse system: inside the particles and in the intergranular channels. Depending on the relations between the corresponding relaxation times and the characteristic time of the problem specified from additional conditions, particular limiting models may be constructed. In [1], under the condition $\tau_1 \ll \tau_2$, approximate equations of the problem are formulated in the region of large and small times for a monodisperse system with spherical particles, and possible applications of the given scheme to practical problems including only one (in the terminology of [8], "equivalent") equation are discussed. Comparison with the experimental data in [11] shows a definite improvement in the agreement between theory and experiment in comparison with other models [1].

The basic advantage of describing the heterogeneous system under the condition that $\tau_1 \ll \tau_2$ reduces to the possibility of constructing closing relations for exchange heat (mass) fluxes by a relatively simple method: requiring that the surface temperature of the particle be equal to the corresponding temperature value in the carrier phase (in the macroscale L), while in the microscale ℓ ($\ell \ll L$), this temperature may be assumed to depend only on the time. This markedly simplifies the calculations in comparison with the various traditional [5, 12] versions of cell models with an isolated particle.

1. General formulas for the heat-transfer characteristics of the particle—disperse-medium system are now obtained with the above-noted relaxational relations, with a negligibly small role of contact resistance to heat transfer. The problem in the region inside the particle is formulated as follows

$$\Delta T = \partial T / \partial t, \quad t = \lambda_2 \tau / d_2 c_2 l^2,$$

(2)

$$T|_{t=0} = T_0 (x, y, z),$$

$$T|_s = T_1 (t),$$

(3)

where $l = V^{1/3}$; V is the particle volume; s is its surface. Laplace transformation with respect to the time is now applied to Eq. (2) and the boundary condition in Eq. (3)

$$\Delta T^* = p T^* - T_0, \quad T^*|_s = T_1^* (p),$$

(4)

where p is the variable of Laplace transformation, and an asterisk denotes transformed quantities. The Green's function $G_1(z, \xi)$ is determined for the Dirichlet problem in the region inside the particle, and expressed in terms of the orthonormalized family of functions $v_i(z)$, which satisfy the equation

$$\Delta v_i + \mu_i v_i = 0, \quad \mu_i = \text{const} > 0, \quad i = 1, 2, \ldots, \tag{5}$$

with a zero value of v_i at the contour of the region. Here and below, the vector symbols such as z denote, for the sake of brevity, a set of three variables: x, y, z for z; ξ, η, ζ for ζ, and so on. As is known [13], G_1 is expressed in terms of the family v_i by the formula

$$\tag{6}$$

$$G_j(z, \xi) = \sum_{i=1}^{\infty} \frac{v_i(z) v_i(\zeta)}{(\mu_i)^j}$$

where j = 1.

Series expansion of the solution of Eq. (4) in terms of v_i and introducing the notation dz = dxdydz gives the following result, using the orthonormalization and completeness of the system v_i

$$\tag{7}$$

$$T^* = \sum_{i=1}^{\infty} \frac{\beta_i v_i(z)}{\mu_i + p} + T_1^* \sum_{i=1}^{\infty} \frac{\mu_i \gamma_i v_i(z)}{\mu_i + p},$$

where

$$\gamma_i = \int_V v_i(z)\, dz, \quad \beta_i = \int_V T_0(z) v_i(z)\, dz. \tag{8}$$

Below, the symbol V for integration over the whole volume of the particle is omitted, and the notation $\oint$ is used for the surface integral over the whole surface.

Introducing the mean temperature $T_2 = \langle T \rangle = \int T(z)dz$ (remember that the particle volume is normalized to unity here) gives

$$T_2^* = T_1^* \Phi^* + B^*,$$

$$\Phi^* = \sum_{i=1}^{\infty} \frac{\mu_i \gamma_i^2}{\mu_i + p}, \quad B^* = \sum_{i=1}^{\infty} \frac{\beta_i \gamma_i}{\mu_i + p}. \tag{9}$$

The heat flux to the particle is determined by the expression

$$Q^* = \oint \frac{\partial T^*}{\partial n}\, ds = \int \Delta T^* dz = p T_2^* - \langle T_0 \rangle = \tag{10}$$

$$= p T_1^* \Phi^* - K^*, \quad K^* = \langle T_0 \rangle - p B^* = \sum_{i=1}^{\infty} \frac{\mu_i \beta_i \gamma_i}{\mu_i + p},$$

which is based on the Gauss—Ostrogradskii theorem and the formula

$$\langle T_0 \rangle = \sum_{i=1}^{\infty} \beta_i \gamma_i.$$

Inverse transformation in Eqs. (9) and (10) gives

$$T_2 = \int_0^t \Phi(t - \xi) T_1(\xi)\, d\xi + B(t), \tag{11}$$

$$Q = \int_0^t T_1(t - \xi) \frac{\partial \Phi}{\partial \xi}\, d\xi + T_1 - K(t) = \int_0^t \Phi(t - \xi) \frac{\partial T_1}{\partial \xi}\, d\xi +$$

$$+ \Phi(t) T_1|_{t=0} - K(t). \tag{12}$$

Substituting the result for Q into the balance equation for the material in the continuous phase, one equation for T_1 is obtained

$$\varepsilon d_1 c_1 \left(\frac{1}{\tau_2} \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) T_1 = \lambda_* \Delta T_1 - \frac{(1 - \varepsilon) \lambda_2}{l^2} \left[T_1 - K(t) + \right.$$

$$\left. + \int_0^t T_1(t - \xi) \frac{\partial \Phi}{\partial \xi}\, d\xi \right], \quad t = \frac{\tau}{\tau_2}, \quad \tau_2 = \frac{d_2 c_2 l^2}{\lambda_2}. \tag{13}$$

Equation (13) has the structure of an equation with "memory" expressed by the integral term. The influence of the initial function T_0 is taken into account by the term K(t). This equation is regarded as macroscopic. Simpler relations corresponding to large and small time values in describing the process by Eq. (13) are now obtained.

2. Consider Larger Times. In obtaining approximate equations, it is expedient to work with Laplace-transformed quantities and to construct expansions in terms of the parameter p. Introducing the functions w_j and the constants n_j

$$w_j(z) = \int G_j(z, \zeta) \, d\zeta, \quad n_j = \int w_j(z) \, dz, \quad j = 1, 2, \ldots \tag{14}$$

the expansion of Φ^* takes the form

$$\Phi^* = 1 + \sum_{j=1}^{\infty} (-1)^j n_j p^j. \tag{15}$$

Then, inverse transformation is undertaken in Eq. (13), and a finite number of terms is retained in the series in Eq. (15); this corresponds to the method [8] of obtaining equivalent equations. There is no need here for expansion of K^* in Eq. (10) similar to Eq. (15), since the equivalent equation is constructed for times of the order of $t \gg \mu_1^{-1}$ and the set of exponents obtained in the transformation of K^* shows that, at such times, $K(t)$ is practically zero. Thus

$$[\varepsilon d_1 c_1 + (1 - \varepsilon) d_2 c_2] \frac{\partial T_1}{\partial \tau} + \varepsilon d_1 c_1 \mathbf{u} \cdot \nabla T_1 = \lambda_* \Delta T_1 -$$

$$- (1 - \varepsilon) d_2 c_2 \sum_{j=2}^{N} (-1)^j n_{j-1} \tau_2^{j-1} \frac{\partial^j T_1}{\partial \tau^j}. \tag{16}$$

As is evident from Eq. (16), n_j must be determined to formulate the problem of heat and mass transfer in a heterogeneous medium consisting of particles of identical size and shape. The functions w_j are determined recurrently from the equation

$$\Delta w_j = -w_{j-1}, \quad j = 1, 2, \ldots; \quad w_0 = 1, \tag{17}$$

as readily follows from Eqs. (5) and (6). At the boundary of the region, w_j vanishes. The relation

$$G_j(z, \zeta) = \int G_i(z, r) G_{j-i}(r, \zeta) \, dr, \tag{18}$$

which resembles, in external form, the equation of the theory of Markov processes [14], may prove useful here. Hence it follows that

$$w_j(z) = \int G_i(z, r) w_{j-i}(r) \, dr, \tag{19}$$

$$n_j = \int w_i(r) w_{j-i}(r) \, dr. \tag{20}$$

The functions w_j are superharmonic ($\Delta w_j \leq 0$), and therefore exceed the solution of the Laplace equation with the same boundary conditions everywhere inside the region. In particular $w_1 \geq 0$ and, since the case $w_1 \equiv 0$ is impossible, max $w_1 = w_* > 0$. It then follows from Eq. (19) by induction that max $w_j \leq w_*^j$ (the function $G_1 \geq 0$); hence an inequality is obtained for n_j: $n_j \leq w_*^j$. This guarantees the convergence of the series in Eq. (15) when $|p| < w_*^{-1}$. Nevertheless, the accurate value for the radius of convergence μ_1 follows from the representation of Φ^* in the form of a sum of simple fractions in Eq. (9).

It follows from Eq. (20) that, knowing k functions w_j, 2k coefficients n_j may be calculated. An exhaustive (fundamentally) solution of the problem may be obtained, knowing the Green's function $G_1(z, \zeta)$; however, finding this function for regions of complex form is associated with certain difficulties. It is simpler to investigate the solution of Eq. (17), the more so in that, in practice, one or two functions w_j may be required to find n_j, i.e., to construct the equivalent equation. Taking into account that n_j is an integral characteristic of the solution, well-developed approximate approaches which give completely acceptable accuracy for integral relations of the solution may be used [15].

The problem for finding w_j coincides with the problem of determining the time to reach the absorbing boundary for a particle moving in the region by the law of Markov processes, with a zero drift coefficient and constant diffusion coefficient [14]. Then n_1 is the mean time to reach the boundary of the region for a particle somewhere inside it (at point z). Apart from some factor, the recurrence formulas in Eq. (17) coincide with the dependences for the "moments" of corresponding order for the mean time in which a particle reaches the boundary of the region from the given internal point. Here n_j corresponds to the mean square, cube, and so on of the time of particle absorption at the boundary. This interpretation of the coefficients in the equivalent Eq. (16) is especially useful and clear in problems of mass transfer in porous systems in the presence of stagnant zones, where the impurity particle again moves in accordance with the above comments and with some correction of the boundary conditions. A probabilistic approach [16] is used here to describe the propagation of the particle in a porous medium, and the presence of stagnant zones leads to correction of the model [16] either by the traditional method — the system in Eq. (1) — or by postulating an equation with memory

of the form in Eq. (13), where the integral term is interpreted in relation to the residence time in the stagnant zone, and so on. In fact, Eq. (1) may also be reduced to a single equation by expressing T_2 in terms of T_1 from the second equation in the form of an integral relation and substituting the result into the first relation of Eq. (1). Thus, conversion to an equivalent equation containing terms up to $\partial^2 T_1/\partial r^2$ (inclusive) corresponds to a rough description of the mass-transfer interaction of the flow section with the stagnant zones, when the distribution function of the particle exit time from the stagnant zone is replaced by its mean value. Note that the integrodifferential equations for mass transfer in a randomly inhomogeneous porous medium were introduced in [17], for example. Note also that relaxation of the mass (temperature) transfer in the basic flow over time occurs in heterogeneous systems; this is associated, to a known extent, with variability of the coefficient λ_* [5]. This may lead to a term $\partial^2 T_1/\partial r^2$ in the basic equation of opposite sign to the term with $\partial^2 T_1/\partial r^2$ due to the presence of the stagnant zones [11]; this may influence the type of equivalent equation. In a porous randomly inhomogeneous medium, it has been noted [18] that the type of transfer equation itself, with the corresponding detailed description (second-order equation), is a relative concept.

3. Consider some examples. The simplest is a spherical particle. The formula for Φ^* was in fact obtained in [19]

$$\Phi^* = 3\,(\sqrt{p}\,\mathrm{cth}\,\sqrt{p} - 1)/p.$$

Expansion of the hyperbolic cotangent in simple fractions [20] gives the representation in Eq. (9) for Φ^*, and the series in powers of p [21]

$$\Phi^* = 6 \sum_{n=1}^{\infty} (p + \pi^2 n^2)^{-1} = 1 + 12 \sum_{n=1}^{\infty} 2^{2n}B_{2n+2}p^n/(2n+2)! \tag{21}$$

allows n_j to be determined: $n_j = (-1)^j 12 \cdot 2^{2j}B_{2j+2}/(2j+2)!$, where B_{2n} is a Bernoulli number.

As the second example, consider a particle of ellipsoidal form $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$, where a, b, c are dimensionless parameters normalized, like x, y, z, to the particle volume ($\pi abc = 1$). The solution of the equation for w_1 takes the form

$$w_1 = A\,(1 - x^2/a^2 - y^2/b^2 - z^2/c^2), \quad A = 0.5/(1/a^2 + 1/b^2 + 1/c^2). \tag{22}$$

From Eq. (20), n_1 and n_2 are found

$$n_1 = \int w_1(z)\,dz = 8A/15, \quad n_2 = \int w_1^2(z)\,dz = 32A^2/105.$$

The function w_2 is sought in the form

$$w_2 = (\alpha x^2 + \beta y^2 + \chi z^2 + \delta)\,w_1,$$

which is substituted into Eq. (17) with j = 2 to give $\delta = A$

$$\beta = \frac{A}{b^2}\left[\frac{(b^2 - a^2)}{8 + a^2} + \frac{(b^2 - c^2)}{8 + c^2} - 1\right]\left\{2 + \frac{(8 + b^2)[64 + 5a^2c^2 + 24\,(a^2 + b^2)]}{b^2\,(8 + a^2)(8 + c^2)}\right\}^{-1}. \tag{23}$$

The coefficients α and χ are determined from Eq. (23) with the substitution $b \leftrightarrow a$ and $b \leftrightarrow c$, in view of the symmetry of the problem. The functions w_j with $j \geq 3$ may be found analogously.

As another example, consider particles in the form of a circular cylinder. Solution of Eq. (17) with j = 1 by the Fourier method and calculation of n_1 from Eq. (20) with j = 1, i = 1 gives the expression

$$\frac{n_1}{a^2} = \frac{1}{8} - \frac{8}{v}\sum_{n=1}^{\infty}\frac{\mathrm{th}\,(\gamma_n v/2)}{\gamma_n^5} = \frac{v^2}{6} - $$

$$- \frac{16v^3}{\pi^5}\sum_{n=0}^{\infty}\frac{I_0'\,[\pi\,(2n+1)/v]}{I_0\,[\pi\,(2n+1)/v](2n+1)^5}, \tag{24}$$

where $\nu = h/a$; $I_0(z)$ is a modified Bessel function; γ_n are the roots of the equation $J_0(\gamma) = 0$; h, a are the cylinder height and base radius normalized to its volume ($\pi a^2 h = 1$). From the second series in Eq. (24), taking account of the asymptotic properties of the Bessel functions [21], it follows that $n_1/a^2 \simeq \nu^2/6$ when $\nu \to 0$. Analogously, it follows from the first series that

$$n_1/a^2 \simeq 1/8 - 0.10124/v, \quad v \to \infty. \tag{25}$$

The function $8n_1/a_2$ is plotted in Fig. 1. Equation (25) may also be used outside the range in Fig. 1.

Note that, on filling with cylindrical grains, they may be in contact over relatively large surfaces, and then the "contact resistance" must be taken into account.

4. Now consider the case of small times: $\tau_1 \leq \tau \leq \tau_2$. In this case, it is again expedient to use the expansion of Laplace-transformed relations, but only in the region of large p. The expressions obtained earlier for Q^*, Φ^*, and other functions are inexpedient. It is simpler to proceed directly from the analysis of Eq. (4). At small times, the process will only occur intensively in the wall region of the particle. Far from the boundary, the initial temperature distribution is retained. To simplify the calculations, $T_0 \equiv 0$ is assumed.

First, the boundary of the region is assumed to be a smooth surface with a minimum radius of curvature of the order of the particle size. In local consideration of the problem, it is expedient to introduce the "internal" [22, 23] coordinates $\zeta = z \cdot \sqrt{p}$ ($p \to \infty$). In the vicinity of the coordinate origin $\zeta = 0$, the equation of the boundary of the region is written in the form $\zeta = (P\xi^2 + R\eta^2)/\sqrt{p}$, where $P = 0.5\partial^2 z/\partial x^2$; $R = 0.5\partial^2 z/\partial y^2$ when $x = 0$, $y = 0$, $z = 0$. The solution of Eq. (4) is sought in the form

$$T^* = T_1^*(p)[E_0(\zeta) + p^{-1/2}E_1(\zeta) + \ldots]. \tag{26}$$

The following problems are obtained for E_0 and E_1

$$\Delta E_0 = E_0, \quad E_0|_{\zeta=0} = 1, \quad E_0|_{\zeta \to \infty} \to 0, \tag{27}$$

$$\Delta E_1 = E_1, \quad E_1|_{\zeta=0} = -(P\xi^2 + R\eta^2)\partial E_0/\partial\zeta|_{\zeta=0}, \quad E_1|_{\zeta \to \infty} \to 0 \tag{28}$$

in realizing the perturbation method [22, 23] with respect to $1/\sqrt{p}$. The problem in Eq. (27) has the solution $E_0 = \exp(-\zeta)$. This allows Eq. (28) to be divided into two, by representing E_1 as the sum of solutions depending on ζ, ξ and ζ, η, respectively. Factors of convergence $\exp(-\delta\xi)$, $\exp(-\delta\eta)$ may be introduced in the boundary condition with $\zeta = 0$, passing subsequently to the limit as $\delta \to 0$, for convenience in realizing integral Fourier transformations in solving the problems obtained. After these manipulations, the following expression is obtained for the heat flux, which is the quantity of interest here

$$\partial E_1/\partial\zeta|_{\zeta=0} = P(1 - \xi^2) + R(1 - \eta^2),$$

and hence the local heat-flux density may be calculated for two approximations with respect to $1/\sqrt{p}$

$$q^* = -\sqrt{p}\left(\frac{\partial E_0}{\partial\zeta}\bigg|_{\xi=0} + \frac{1}{\sqrt{p}}\frac{\partial E_1}{\partial\zeta}\bigg|_{\xi=0}\right)T_1^* = T_1^* \cdot (\sqrt{p} - \varkappa), \tag{29}$$

where $\varkappa = P + R$ is the surface curvature as a function of a point on it. integrating Eq. (29) over the whole particle surface and substituting the result into the heat-balance equation in the first phase, the desired equation is obtained after Laplace transformation

$$\varepsilon d_1 c_1 \left(\frac{\partial}{\partial\tau} + \mathbf{u} \cdot \nabla\right)T_1 = \lambda_* \Delta T_1 - \frac{(1 - \varepsilon)S\lambda_2}{V}\left\{\sqrt{\frac{d_2 c_2}{\pi\lambda_2}} \times\right.$$

$$\left. \times \left[\int_0^\tau \frac{\partial T_1}{\partial\xi}\frac{d\xi}{\sqrt{\tau - \xi}} + \tau^{-1/2}T_1|_{\tau=0}\right] - \frac{T_1}{S}\oint\varkappa(s)\,ds\right\}, \tag{30}$$

where S is the surface area of the particle. In [1], in the equation analogous to Eq. (30), the lower limit of the integral is $-\infty$, which corresponds to a sufficiently large value of the time after the onset of the process. The second term in the square brackets may be neglected here. The last term on the right-hand side of Eq. (30) refines the equation of [1], since it takes account of the next approximation.

The derivation of Eq. (30) assumes a smooth surface surrounding the particle. Heat and mass transfer in packing consisting of particles with sharp changes in the boundary surface — for example, particles in the form of cylindrical tablets — is also of interest. Various features at the boundary are possible.

Consider the sufficiently typical case when, in the vicinity of the given point, the particle surface may be approximated, with adequate accuracy, by a wedge with aperture angle 2ψ in the "internal coordinates" introduced above. Confining analysis to the first approximation, the subscript 1 on E_1 is omitted for the sake of brevity, and the corresponding problem is considered in a cylindrical coordinate system (ρ, φ)

$$\rho\frac{\partial}{\partial\rho}\rho\frac{\partial E}{\partial\rho} + \frac{\partial^2 E}{\partial\varphi^2} = \rho^2 E, \quad E|_{\varphi=0;\,2\psi} = 1, \quad E|_{\substack{\rho \to \infty \\ \rho \to 0}} \to \text{bounded} \tag{31}$$
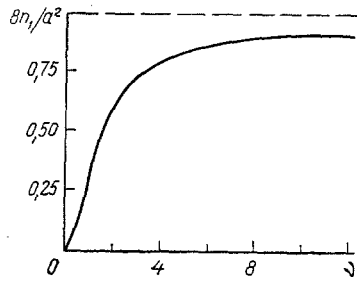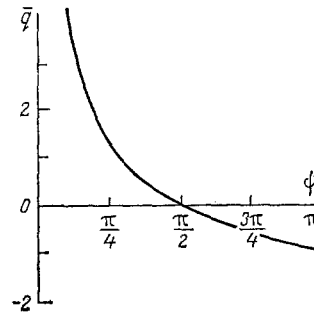
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Fig. 1   Fig. 2

Fig. 1. Dependence of the parameter $8n_1/a^2$ on the form of the cylinder.

Fig. 2. Additional flux at two faces of the wedge as a function of the
semivertex angle.

The function E satisfying Eq. (31) is found using the Kontorovich—Lebedev integral transformation [24], in the
form

$$E(\rho, \varphi) = \frac{2}{\pi} \int_0^\infty \frac{\mathrm{ch}\,(\pi x/2)}{\mathrm{ch}\,(\psi x)} \mathrm{ch}\,[x(\psi - \varphi)]\,K_{ix}(\rho)\,dx, \qquad (32)$$

where $K_{ix}(\rho)$ is the MacDonald function; i is the imaginary unity. The additional heat flux due to the presence of the
wedge singularity in comparison with the flux given by Eq. (27) is of basic interest here. Accordingly, the function
$E_0(\zeta) = \exp(-\rho \sin \varphi)$ is written in the form of a Kontorovich—Lebedev transformation [24]

$$\exp(-\rho \sin \varphi) = \frac{2}{\pi} \int_0^\infty K_{ix}(\rho)\,\mathrm{ch}\left[x\left(\frac{\pi}{2} - \varphi\right)\right]dx,$$

and subtracted from Eq. (32), and then a formula analogous to Eq. (10) is used in a sector with aperture angle ψ. At the
boundary $\varphi = 0$, there is then a difference between the fluxes given by Eqs. (32) and (27), i.e., the difference required,
and at the boundary $\varphi = \psi$ the flux associated with the function E is zero and the "parasitic" flux due to E_0 must be
eliminated in order to obtain the true value for the additional flux at one edge of the wedge. Using the relation

$$\int_0^\infty y K_{ix}(y)\,dy = \frac{\pi x}{2\,\mathrm{sh}\,(\pi x/2)}$$

and employing the program of actions outlined above, an expression is obtained for the additional flux $\overline{q}$ per unit length
of the rib

$$\overline{q}(\psi) = \int_0^\infty \frac{\mathrm{sh}\,[(\pi/2 - \psi)\,x]\,dx}{\mathrm{sh}\,(\pi x/2)\,\mathrm{ch}\,(\psi x)}. \qquad (33)$$

A graph of $\overline{q}$ is shown in Fig. 2. At small ψ, the following asymptotic expansion may be obtained

$$\overline{q} = \frac{\ln 2}{\psi} - \sum_{n=1}^\infty \frac{4^{2n}(4^n - 1)B_{2n}B_n}{2n\,(2n)!}\psi^{2n-1}, \quad \psi \to 0,$$

complementing Fig. 2. To take account of the contribution of the second face of the bihedral corner, $\overline{q}$ must be
multiplied by two. Taking account of the "rib effect" leads to a term of the order of unity with respect to p as p → ∞
in the expression for the flux, which adds the expression $2\overline{q}LT_1/S$ to the last term on the right hand side of Eq. (30),
where L is the length of the rib with the given constant aperture angle 2ψ, or if this angle varies slowly along the rib:
$(2T_1/S)\int_L \overline{q}[\psi(l)]dl$. For example, for a circular cylinder, this correction is $(\overline{q} = 2/\pi)8T_1/[\pi(a + h)]$, where a is the radius;
h is the cylinder height, and two circumferences of length $2\pi a$ are taken into account. The contribution of the term
containing the mean surface curvature is $T_1 h/[2a(a + h)]$ here.

Singular points in the form of conical points, trihedral angles, and other similar types may be encountered at the
particle surface. It is readily evident that the additional contribution from these points is of the next order in p —

$[O(1/\sqrt{p})]$ as $p \to \infty$ — and therefore may be neglected within the framework of Eq. (30). For example, the trihedral angle formed by three mutually perpendicular planes adds a term $(12/\pi)T_1^*/\sqrt{p}$ to the heat flux.

5. The construction of equivalent equations may also be considered from the viewpoint of the small-perturbation method [22, 23]. As noted in [8], the region of applicability of the equivalent equation corresponds to a small value of the parameter r_2/r_*. This parameter may be taken as the basis in constructing a solution of the general heat- and mass-transfer problem by the perturbation method on the basis of equation of the type in Eq. (13). In the first approximation, a parabolic single-phase equation will be obtained here, and in the next approximation, instead of an elliptical (equivalent) equation, again a parabolic equation, but one which is inhomogeneous, with a source depending on the first approximation. The accuracy of this construction corresponds to the accuracy of the elliptical equation. The possible appearance of nonuniformity with respect to the coordinates and the time [8] is eliminated by the traditional construction of a uniformly applicable expansion [22, 23]. The given approach permits the elimination of the small parameter in the higher derivatives in the elliptical equation, simplifies the issue of correct formulation of the problem, and permits a more well-founded approach to the formulation of the initial conditions, determining them by means of matching with the internal solution at times of the order of r_2. Without going into these questions in more detail, it will be noted here that this construction is similar to the problem of [9], to a known extent. Note also that the elliptical equivalent equation is analogous in its derivation to the well-known Ozen equation in considering the flow of a viscous liquid around a particle at low Reynolds numbers. The above questions (with replacement of the time coordinate by the spatial coordinate) also appear there [23].

NOTATION

c, specific heat; d, density; T, temperature; T_0, initial temperature in particle; u, velocity of transfer (filtration); x, y, z, Cartesian coordinates; ε, proportion of continuous phase in unit volume of packing; λ_*, effective thermal conductivity in continuous medium; λ_2, thermal conductivity in disperse phase; μ_i, eigenvalues of the Dirichlet problem in the region inside the particle; ξ, η, internal coordinates; σ, heat-transfer coefficient; r, time; r_1, r_2, relaxation times in continuous and disperse phases, respectively; r_*, characteristic time of problem. Indices: 1, continuous phase; 2, disperse phase.

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MASS TRANSFER BETWEEN PHASES AND WATER TRANSPIRATION IN

A MEDIUM HAVING DOUBLE POROSITY

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Two limiting models are proposed for exchange involving porous granules containing water in the condensed and vapor states. The water uptake rate by plants can be related to the water content.

A model exists for water transport in soil that incorporates plant root transpiration, which leads to a quasilinear parabolic equation for the water content, but it involves major empirical elements and far from always correctly reflects the actual processes [1]. This is due in part to the transport mechanisms assumed and in part to the description of the plant water uptake. The first aspect has been discussed in [2], where water filling was considered for a two-pore medium that simulated a granulated soil. The second amounts to determining the transpiration rate as a function of soil water content.

Various forms of that relationship have been proposed [3-6]. It is usually approximated as a kinked line consisting of several straight sections, with the coefficients taken as certain universal parameters completely determined by root mass density and plant physiology. In fact, that approach is essentially incorrect because the transpiration rate is dependent not only on the water content in the soil but also on the transport rate directly to the roots. The latter varies with the soil structure. Therefore, for a given water content, the transpiration rates for given plants will be dependent on the water transport in the soil, i.e., one cannot consider the process as universal.

1. We represent the soil as consisting of contacting porous granules [2], which for simplicity we take as identical spheres. The plant roots usually lie in the gaps between the granules and absorb water vapor [3]. We assume that condensed water occurs only within the granules, and the water enters the space between them by evaporation and transport in the granules. The rates are usually much less than the mixing rates in the intergranules space on scales of the order of the characteristic microstructure length, so the vapor concentration c can be taken as homogeneous. It is realistic to assume that the rate-limiting step in transpiration is water absorption at the root-surfaces, not transport in the gas. One then represents the uptake as a first-order reaction, and the mass of water absorbed in unit volume of soil in unit time is kc, where k is the product of the specific area of the active root-system surface and the rate constant for the reaction, which may be considered as known.

Several physical mechanisms are involved in water transport within the granules [1, 7, 8]. Heuristically, one can distinguish transport in the condensed state by capillary impregnation, the motion of thin liquid films diffusion in sorbed layers, and so on, as well as diffusion of the evaporating water in the pore space not filled by liquid.

The simultaneous description of those processes is exceptionally complicated, as is familiar from drying theory [8]; to consider the essence, we discuss only simple models, which correspond essentially to different transport rates in the condensed and vapor states subject to some simplifying assumptions.

2. Let the condensed-water transport rate be much less than the vapor rate in the gas. Then we get a model for the evaporation front r = R(t) in each granule, which separates the region r < R, in which part κ of the pore space is filled by condensed water, from the part containing water only as vapor [9]. The front as a zero-thickness surface is an idealization, because there are size differences in the capillaries and pores, and saturation pressure differences over the corresponding menisci, together with transport of condensed or sorbed water, so the front is diffuse and there is a finite-thickness inhomogeneous zone. However, the model with a step change in water content a the front is acceptable if that thickness is much less than the granule radius R_0.